	The Straight Line	**Notes**
	Distance formula	
	Mid-point formula	
	Gradient formula	
	Gradient: $m = \tan\theta$	
	Parallel lines: equal gradients	
	Perpendicular lines: product of gradients = –1	
	Gradients of lines parallel to x and y axes	
	Equations of lines parallel to x and y axes	
	Equation of a straight line: $y = mx + c$	
	Equation of a straight line through a point (a, b) with gradient m	
	Find points of intersection	
	Know Median of a triangle	
	Know Altitude of a triangle	
	Know Perpendicular bisector of a line	
	Composite and Inverse Functions	
	Know meaning of domain	
	Know meaning of range	
	Finding expressions for related functions: $f(x+1)$ or $f(3x)$	
	Evaluating functions: e.g. $f(2)$	
	Composite functions: $f(g(x))$; $g(f(x))$	
	Finding inverse of functions	
	Algebraic Functions and Graphs	
	Completing the square	
	Simple case: $x^2 + 2x - 5$	
	Common factor type: $3x^2 + 6x - 7$	
	Negative common factor: $3 - 8x - 2x^2$	
	Maximum and minimum values from completing the square	
	Sketching graphs of related functions: $y = -f(x)$, $y = f(-x)$	
	Sketching graphs of related functions: $y = f(x \pm k)$, $y = f(x) \pm k$	
	Know special logs: $\log_a 1 = 0$ and $\log_a a = 1$	
	Trigonometric Functions and Graphs	
	Changing between radians and degrees π radians = 180°	
	Common values of radians ~ degrees e.g. $\pi/6 = 30°$	
	Exact value table for sin, cos, tan of 30°, 45°, 60° (surds)	
	Max and min values of trig functions	look where sin and cos are 1 or –1

	Using All Sinners Take Care	
	Recognising Trig graphs: $y = a \sin bx$, $y = a \cos bx$ (± constant)	
	Sketching Trig graphs: $y = a \sin bx$, $y = a \cos bx$ (± constant)	
	Solving Trig Equations: - *always aim to get* $\sin x = constant$	
	Type 1: $2 \sin x = 1$	
	Type 2: $\sqrt{2} \sin x + 1 = 0$	
	Type 3: $\sin 3x = -1$	
	Type 4: $2 \sin^2 x = 1$	
	Type 5: $4 \sin^2 x + 11 \sin x + 6 = 0$	
	Type 6: $\sin^2 x - \cos^2 x = 1$	
	Type 7: $\sin (2x - 20°) = 0.5$	
	Introduction to Differentiation	
	Rules for differentiation:	
	Constant a	
	Power of x x^3	
	Constant times power of x ax^4	
	sum or difference $3x^2 - 5x^3$	
	Negative indices x^{-3}	
	Fractional indices $x^{-4/5}$	
	Fractions $\frac{3}{x^2}$	Straight line form
	Roots and Powers $\sqrt[3]{x^2}$	Straight line form
	Fraction expression $\frac{3x^4 + 5}{x}$	Straight line form
	Rules of indices	
	Meaning of negative indices	
	Meaning of fractional indices	
	Finding gradient of tangent to: $y = f(x)$ at $P(a, b)$	
	Finding equation of tangent to: $y = f(x)$ at $P(a, b)$	
	Finding point on a curve where tangent has a given gradient	
	Using Differentiation	
	Using table of signs – to determine nature of stationary point	
	Using velocity and acceleration as derivatives	
	Sequences	
	Using a recurrence relation to generate terms: $u_0, u_1, u_2, \ldots$..	
	Forming a recurrence relation	
	The linear recurrence relation: $u_{n+1} = m\, u_n + c$	
	Special sequences: when $m = 1$ arithmetic sequence	
	when $c = 0$ geometric sequence	
	Limit of a recurrence relation: If m is fractional: $\mathbf{L = c/(1 - m)}$	

Polynomials		
	Nested or synthetic division: dividing by (x – h)	
	Dividing by (x + h) or (2x + h)	
	Write down the quotient and remainder	
	Remainder Theorem: Remainder is f(h) when dividing by x - h	
	Factor Theorem: If f(h) = 0 then (x – h) is a factor	
	Finding factors of polynomials	– look at factors of constant
Quadratic Theory		
	Solving quadratic equations: 4 methods	
	Graphically	
	Factorisation	
	Trinomial eg $x^2 + 5x + 6 = 0$ $(x + 3)(x + 2)$	
	Common Factor eg $x^2 + 5x = 0$ $x(x + 5) = 0$	
	The Quadratic formula	
	Completing the Square	
	Using the discriminant to determine nature of roots: $b^2 – 4ac$	
	= 0 (equal, real) >0 (real, distinct) <0 (no real roots)	
Integration		
	Rules of integration – reverse of differentiation	Straight line form
	Increase the index, Divide by the new index	
	Do not forget the constant of integration	
	Finding equation of a curve from gradient function and a point	Integrate and substitute to find c
	Integration of fractional and negative indices	Straight line form
	The area under a curve – defining as a definite integral	
	Write down definite integrals (representing area under a curve)	
	Evaluating definite integrals	
	Calculating area under a curve	
	Meaning of negative area below x-axis	
	Composite areas	
	Area between two curves	
Calculations in 2 and 3 dimensions		
	SOH-CAH-TOA	
	Sine Rule	
	Cosine Rule	
	Area of triangle - 2 formula ~ **½ base x height** : **½ a b sin C**	
	Related angles: sin(180 – A) = sin A sin (-A) = - sin A	
	Related angles: cos(180 – A) = -cos A cos (-A) = cos A	
	Related angles: cos (90 - A) = sin A sin (90 - A) = cosA	
	Trigonometric Proofs	
	3D – angle between a line and a plane	

	3D – angle between two planes	
	3D – Face diagonal	
	3D – Space diagonal	
Compound Angle Formula		
	Reminder of: sin A / cos A = tan A	
	Reminder of: $\sin^2 A + \cos^2 A = 1$	
	cos (A + B) = cos A cos B – sin A sin B	
	cos (A – B) = cos A cos B + sin A sin B	
	sin (A + B) = sin A cos B + cos B sin A	
	sin (A – B) = sin A cos B – cos B sin A	
	sin 2A = 2 sin A cos A	
	$\cos 2A = \cos^2 A – \sin^2 A$	
	$\cos 2A = 2 \cos^2 A - 1$ $\cos 2A = 1 – 2 \sin^2 A$	
	$\cos^2 A = ½ (1 + \cos 2A)$ $\sin^2 A = ½ (1 - \cos 2A)$	
	Finding exact values for sin 15°, cos 75° etc.	
	Proving identities e.g. prove: $\sin 3A = 3 \sin A – 4 \sin^3 A$	
	Solving trig. equations:	
	Type 1: cos 2x + cos x + 1 = 0	solve a quadratic using common factor
	Type 2: cos 2x + cos x + k = 0 where k ≠ 1	solve a quadratic with two brackets
The Circle		
	Use form of equation of circle – centre O, radius r	$\mathbf{x^2 + y^2 = r^2}$
	Given equation – write down radius	
	Given radius – write down equation	
	Check if a point lies on, inside or outside the circle	Compare distance from origin with radius
	Condition for (p, 2) to lie on circle	(p, 2) must satisfy equation of circle
	Equation of circle – centre C(a, b) and radius r	$\mathbf{(x – a)^2 + (y – b)^2 = r^2}$
	Given equation – write down radius and co-ordinates of centre	
	Given radius and co-ordinates of centre – write down equation	
	Two circles touch – Distance between centres = sum of radii	
	Finding common point where two circles touch	Use proportion on line joining centres
	Angle in a semi circle	is a right angle = 90°
	Properties of perpendicular bisector of a chord	
	Isosceles triangles in a circle	
	Tangent to a circle is at right angles to radius (or diameter)	
	General equation of a circle (expand $(x – a)^2 + (y – b)^2 = r^2$)	$\mathbf{x^2 + y^2 + 2gx + 2fy + c = 0}$
	Knowing that centre is at (–g, –f)	
	Knowing radius is: $\sqrt{(g^2 + f^2 – c)}$	
	Finding equation of tangent to a circle	right angles to radius
	Intersection of lines and circles	simultaneous equations - substitution
	Condition for a line to be a tangent to a circle	1 point of intersection - discriminant

Vectors		
	Write a directed line segment as a column vector	
	Calculate magnitude of a vector |**u**|	
	Adding and subtracting vectors	Add or subtract components
	Multiply by a scalar	Multiply each component by the scalar
	Writing a directed line segment in terms of position vectors	$\overrightarrow{AB} = \underline{b} - \underline{a}$
	Showing 3 points are collinear	Use scalar multiples and **common** point
	Position vector **m** of mid-point of AB	$\underline{m} = \frac{1}{2}(\underline{a} + \underline{b})$
	Find ratio that Q divides PR from co-ordinates of P, Q, R	Find scalar multiple of $\overrightarrow{PQ} : \overrightarrow{QR}$
	Find co-ordinates of Q that divides PR in ratio 2 : 3 (say)	$\frac{\overrightarrow{PQ}}{\overrightarrow{QR}} = \frac{2}{3}$ $3\overrightarrow{PQ} = 2\overrightarrow{QR}$ $3(\underline{q}-\underline{p})=2(\underline{r}-\underline{q}$
	Change between column vector and unit vector form	$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 3\underline{i} - \underline{j} + 2\underline{k}$
	Calculate scalar product **a**.**b** = |**a**| |**b**| cos θ	θ is angle pointing **out** of two vectors
	Calculate scalar product **a**.**b** = $x_1x_2 + y_1y_2 + z_1z_2$	
	Calculate angle between two vectors	$\cos\theta = \frac{\underline{a} \cdot \underline{b}}{\lvert\underline{a}\rvert \lvert\underline{b}\rvert} = \frac{x_1x_2 + y_1y_2 + z_1z_2}{\lvert\underline{a}\rvert \lvert\underline{b}\rvert}$
	Show 2 vectors are perpendicular (at right angles)	Show cos θ = 0 i.e. θ = π/2
	Show 2 vectors are parallel	Show cos θ = 1 i.e. θ = 0
	Using: **a**.**a** = a^2 where a is |**a**|	
	Using: **i**.**i** = **j**.**j** = **k**.**k** = 1	
	Using: **i**.**j** = **i**.**k** = **j**.**k** = 0	
	Use Distributive Law: **a**.(**b** + **c**) = **a**.**b** + **a**.**c**	
Further Differentiation and Integration		
	Differentiate sin x	d/dx (sin x) = cos x
	Differentiate cos x	d/dx (cos x) = - sin x
	Differentiate functions involving multiples of sin x and cos x	d/dx (3sin x – 2cos x) = 3cos x + 2sin x
	Put functions involving sin x, cos x and x^n in straight line form	$\frac{3 + x^2 \cos x}{x^2} \Rightarrow 3x^{-2} + \cos x$
	Using the Chain Rule: If y = f(g(x)) then y = f(u) u = g(x)	dy/dx = dy/du x du/dx
	Using the chain rule for Trigonometric functions	d/dx(sin 2x) = 2 cos 2x
	Integration of standard integral	$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c$
	Integral of Trigonometric functions	
	$\int \cos x \, dx = \sin x + c$	
	$\int \sin x \, dx = -\cos x + c$	
	$\int \cos(ax+b) \, dx = \frac{1}{a}\sin(ax+b) + c$	

	$\int \sin(ax+b)\ dx = -\frac{1}{a}\cos(ax+b) + c$	
	Definite trigonometric integrals	
	$\int_0^{\pi} \sin x\, dx = \left[-\cos x\right]_0^{\pi}$ $= -\cos\pi - (-\cos 0) = -(-1) - (-1) = 2$	
The Exponential and Logarithmic Functions		
	Growth Calculations – e.g. £100 in bank at 12% for n years	Amount = $A(n) = 100 \times 1.12^n$
	Decay calculations – e.g. full tank of 50 litres of petrol evaporates at 20% per week	Amount left $A(n) = 50 \times 0.8^n$
	Linking exponential to the log function	$y = a^x \Leftrightarrow \log_a y = x$
	Write a log in exponential form	$\log_2 8 = 3 \Leftrightarrow 8 = 2^3$
	Write an exponential in log form	$81 = 3^4 \Leftrightarrow \log_3 81 = 4$
	Rules of logarithms	
	$\log_a xy = \log_a x + \log_a y$	
	$\log_a {}^x/_y = \log_a x - \log_a y$	
	$\log_a x^p = p \log_a x$	
	Solving log equations using the above rules	
	Solving exponential equations	Take logs of both sides to exponent base
The Wave Function		
	Express: a sinx + b cos x in the form R cos(x ± a)	
	Express: a sinx + b cos x in the form R sin(x ± a)	
	Finding maxima and minima of: a cos x + b sin x	
	Solving trig equations of the form:	
	sin x + 3cos x + 1 = 0	Use the wave function
	sin 2x + 5cos 2x + 1 = 0	Use the wave function